



A. Factoring out common factors

Find the common factor and take it out.

Example 1: Factor $6x^3 - 4x$. The common factor is $2x$, thus we have $6x^3 - 4x = 2x(3x^2 - 2)$

Example 2: Factor $2x(x - 2) + 3(x - 2)$. We have a linear common factor $(x - 2)$, thus we have $2x(x - 2) + 3(x - 2) = (x - 2)(2x + 3)$

B. Factoring Special Polynomials Forms

<u>Factored Form</u>	<u>Example</u>
<i>Difference of Two Squares</i>	
$a^2 - b^2 = (a + b)(a - b)$	$9x^2 - 4 = (3x + 2)(3x - 2)$
<i>Perfect Square Trinomial</i>	
$a^2 + 2ab + b^2 = (a + b)^2$	$x^2 + 6x + 9 = (x + 3)^2$ where $a = x$ and $b = 3$
$a^2 - 2ab + b^2 = (a - b)^2$	$x^2 - 6x + 9 = (x - 3)^2$ where $a = x$ and $b = 3$
<i>Sum or Difference of Two Cubes</i>	
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	$x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	$x^3 - 8 = x^3 - 2^3 = (x - 2)(x^2 + 2x + 4)$

Note: Remember a , b or both could be represented as a product of other factors or a linear factor, then you have to figure out what is a and b .

Example: $(x + 2)^2 - 16x^4$. Observe that $a = (x + 2)$ and $b = 4x^2$. Applying the Difference of Two Squares formula, we have

$$\begin{aligned} (x + 2)^2 - 16x^4 &= (x + 2)^2 - (4x^2)^2 = [(x + 2) + 4x^2][(x + 2) - 4x^2] \\ &= (x + 2 + 4x^2)(x + 2 - 4x^2) \end{aligned}$$



C. Trinomials with Binomial Factors

To factor a trinomial of the form $ax^2 + bx + c$, use the pattern below

$$ax^2 + bx + c = (\boxed{}x + \boxed{})(\boxed{}x + \boxed{})$$

Factors of a (pointing to the x terms)
Factors of c (pointing to the constant terms)

• **Factoring a Trinomial: Leading Coefficient Is 1**

Since $a = 1$, we have $x^2 + bx + c = (x + \boxed{})(x + \boxed{})$

Factors of c (pointing to the constant terms)

So, we are looking for two factors of c which give us sum of b .

Example: Factor $x^2 - 7x + 12$

PRODUCT $c = 12$		SUM $b = -7$
1	12	13
2	6	8
3	4	7
-1	-12	-13
-2	-6	-8
-3	-4	-7

Table 1

Observe that only factors -3 and -4 work since the product and the sum satisfy $c = 12$ and $b = -7$. Thus, we have $x^2 - 7x + 12 = (x - 3)(x - 4)$

• **Factoring a Trinomial: Leading Coefficient Is Not 1**

Example: Factor $2x^2 + x - 15$

a) Factoring using BOX method

Factors of $ac = 2(-15) = -30$		Sum $b = 1$
1	-30	-29
-1	30	29
2	-15	-13
-2	15	13
3	-10	-7
-3	10	7
5	-6	-1
-5	6	1

Table 2

Factor 1 **Factor 2**



So, two factors that give us the product of -30 and the sum of 1 are -5 and 6 .

Use the following box to set up everything (GCF is Greatest Common Factor).

GCF	First Term	Factor 1	Table 3
GCF	Factor 2	Last Term	

So we have

GCF

x	$2x^2$	$-5x$	$2x - 5$	Table 4
3	$6x$	-15	$2x - 5$	

Notice that after factoring x , we get $2x - 5$ on the first row, and after factoring 3 , we also get $2x - 5$ on the second row. So, we have

$$2x^2 + x - 15 = (x + 3)(2x - 5)$$

a) Factoring by grouping

Using Table 2, we rewrite the middle term as $-5x + 6x$. So we get

$$2x^2 + x - 15 = 2x^2 - 5x + 6x - 15$$

Then we factor the polynomial by grouping: GCF for the first group $2x^2 - 5x$ is x and GCF for the second group $6x - 15$ is 3 . So, we have

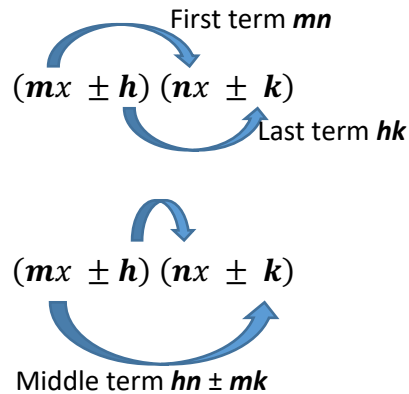
$$2x^2 + x - 15 = 2x^2 - 5x + 6x - 15 = x(2x - 5) + 3(2x - 5) = (2x - 5)(x + 3)$$

b) Factoring using trial and error method

Factors of the first term		Factors of the second term		Table 5
x-term	constant	x-term	constant	
1	2	1	15	
		3	5	



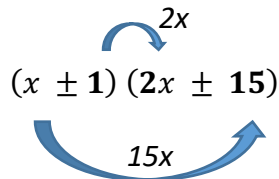
Following rules of multiplication of two factors, we must get the **first term** $2x^2$, the **middle term** x , and the **last term** -15 .



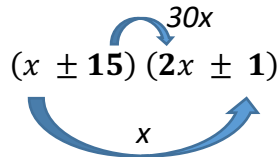
Since we have only two factors for the first term, then we have $m = 1$ and $n = 2$ or vice versa.

For the last term we have two possible factors, 1 and 15 or 3 and 5 , thus we try both possibilities switching factors to get middle term x :

Trying 1 and 15

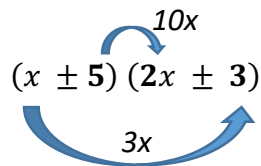


Middle term is $2x \pm 15x$ which doesn't give us x

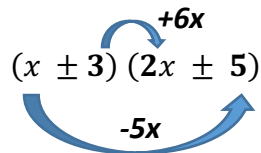


Middle term is $30x \pm x$ which doesn't give us x

Trying 3 and 5



Middle term is $10x \pm 3x$ which doesn't give us x



Middle term is $6x \pm 5x$ which does give us x when $6x$ is **positive** and $5x$ is **negative**:

$$6x - 5x = x$$

Thus we have $2x^2 + x - 15 = (x + 3)(2x - 5)$