A. Factoring out common factors

Find the common factor and take it out.

Example 1: Factor $6x^3 - 4x$. The common factor is *2x,* thus we have $6x^3 - 4x = 2x(3x^2 - 2)$

Example 2: Factor $2x(x-2) + 3(x-2)$. We have a linear common factor $(x-2)$, thus we have $2x(x-2) + 3(x-2) = (x-2)(2x+3)$

B. Factoring Special Polynomials Forms

Factored Form	Example
Difference of Two Squares	
$a^2 - b^2 = (a+b)(a-b)$	$9x^2 - 4 = (3x+2)(3x-2)$
Perfect Square Trinomial	
$a^2 + 2ab + b^2 = (a+b)^2$	$x^2 + 6x + 9 = (x+3)^2 \; where \; a = \; x \; and \; b = 3$
$a^2 - 2ab + b^2 = (a-b)^2$	$x^2 - 6x + 9 = (x-3)^2 \; where \; a = \; x \; and \; b = 3$
Sum or Difference of Two Cubes	
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$	$x^3 + 8 = x^3 + 2^3 = (x+2)(x^2 - 2x + 4)$
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$	$x^3 - 8 = x^3 - 2^3 = (x-2)(x^2 + 2x + 4)$

Note: *Remember a, b or both could be represented as a product of other factors or a linear factor, then you have to figure out what is a and b.*

Example: $(x+2)^2 - 16x^4$. Observe that $a = (x+2)$ and $b = 4x^2$. Applying the Difference of Two Squares formula, we have

$$\begin{aligned}(x+2)^2 - 16x^4 &= (x+2)^2 - (4x^2)^2 = [(x+2) + 4x^2][(x+2) - 4x^2] \\ &= (x+2+4x^2)(x+2-4x^2)\end{aligned}$$

C. Trinomials with Binomial Factors

To factor a trinomial of the form $ax^2 + bx + c$, use the pattern below

$$ax^2 + bx + c = (\underset{}{\overset{\text{Factors of } a}{\square}}x + \underset{\text{Factors of } c}{\square})(\overset{\text{Factors of } a}{\square}x + \underset{\text{Factors of } c}{\square})$$

- **Factoring a Trinomial: Leading Coefficient Is 1**

Since $a = 1$, we have $x^2 + bx + c = (x + \underset{\text{Factors of } c}{\square})(x + \underset{\text{Factors of } c}{\square})$

So, we are looking for two factors of *c* which give us sum of *b*.

Example: Factor $x^2 - 7x + 12$

Table 1

PRODUCT $c = 12$		SUM $b = -7$
1	12	13
2	6	8
3	4	7
-1	-12	-13
-2	-6	-8
-3	-4	-7

Observe that only factors -3 and -4 work since the product and the sum satisfy $c = 12$ and $b = -7$. Thus, we have $x^2 - 7x + 12 = (x - 3)(x - 4)$

- **Factoring a Trinomial: Leading Coefficient Is Not 1**

Example: $Factor\ 2x^2 + x - 15$

a) <u>Factoring using BOX method</u>

Table 2

Factors of $ac = 2(-15) = -30$		Sum $b = 1$
1	-30	-29
-1	30	29
2	-15	-13
-2	15	13
3	-10	-7
-3	10	7
5	-6	-1
-5	6	1
Factor 1	**Factor 2**	

So, two factors that give us the product of *-30* and the sum of *1* are *-5* and *6.*

Use the following box to set up everything (GCF is Greatest Common Factor).

GCF	First Term	Factor 1
GCF	Factor 2	Last Term

Table 3

So we have

GCF

x	$2x^2$	$-5x$	$2x-5$
3	$6x$	-15	$2x-5$

Table 4

Notice that after factoring *x*, we get *2x – 5* on the first row, and after factoring *3*, we also get *2x – 5* on the second row. So, we have

$2x^2+x-15=(x+3)(2x-5)$

a) Factoring by grouping

Using Table 2, we rewrite the middle term as $-5x+6x$. So we get

$2x^2+x-15=2x^2-5x+6x-15$

Then we factor the polynomial by grouping: GCF for the first group $2x^2-5x$ is *x* and GCF for the second group $6x-15$ is *3*. So, we have

$$2x^2+x-15=2x^2-5x+6x-15=x(2x-5)+3(2x-5)=(2x-5)(x+3)$$

b) Factoring using trial and error method

Factors of the first term		**Factors of the second term**	
x-term	constant	x-term	constant
1	2	1	15
		3	5

Table 5

Following rules of multiplication of two factors, we must get the **first term** $2x^2$, the **middle term** x, and the **last term** -15.

First term ***mn***

$$(\boldsymbol{m}x \pm \boldsymbol{h})\,(\boldsymbol{n}x \pm \boldsymbol{k})$$

Last term ***hk***

$$(\boldsymbol{m}x \pm \boldsymbol{h})\,(\boldsymbol{n}x \pm \boldsymbol{k})$$

Middle term ***hn* ± *mk***

Since we have only two factors for the first term, then we have $m = 1 \; and \; n = 2$ or vice versa.

For the last term we have two possible factors, ***1*** and ***15*** or ***3*** and ***5***, thus we try both possibilities switching factors to get middle term ***x:***

Trying 1 and 15

2x

$$(x \pm \mathbf{1})\,(\mathbf{2}x \pm \mathbf{15})$$

15x

Middle term is ***2x ± 15x*** which doesn't give us ***x***

30x

$$(x \pm \mathbf{15})\,(\mathbf{2}x \pm \mathbf{1})$$

x

Middle term is ***30x ± x*** which doesn't give us ***x***

Trying 3 and 5

10x

$$(x \pm \mathbf{5})\,(\mathbf{2}x \pm \mathbf{3})$$

3x

Middle term is ***10x ± 3x*** which doesn't give us ***x***

+6x

$$(x \pm \mathbf{3})\,(\mathbf{2}x \pm \mathbf{5})$$

-5x

Middle term is ***6x ± 5x*** which does give us ***x*** when ***6x*** is **positive** and ***5x*** is **negative**:

6x – 5x = x

+6x

Thus we have $2x^2 + x - 15 = (x+3)(2x-5)$

-5x